

# Boosting:

**A weighted crowd of  
narrowminded  
experts**

Aaron & Dylan

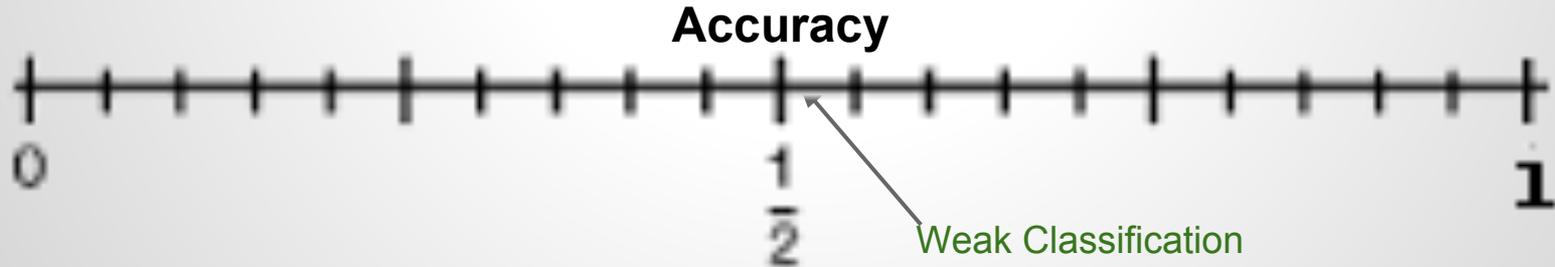


# Boosting Hypothesis (Kearns, Valiant; 1988-89)

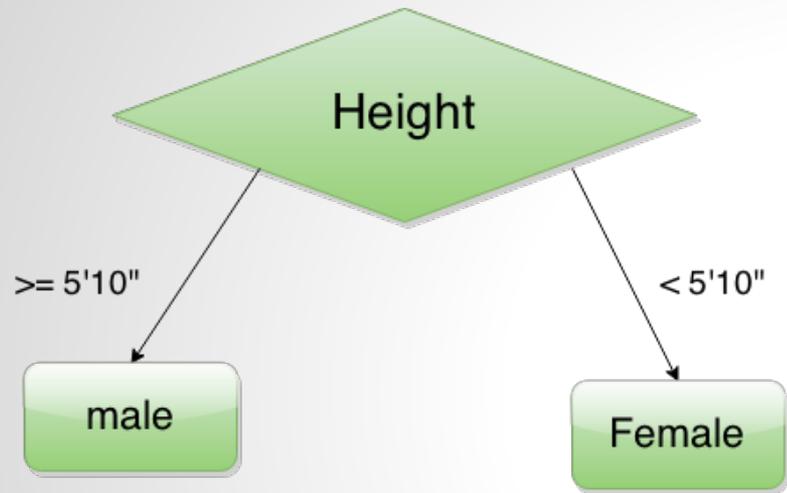
We can make a strong classifier (arbitrarily well at classification) from a collection of weak classifiers (somewhat better than random guess).

## Weak Classifiers

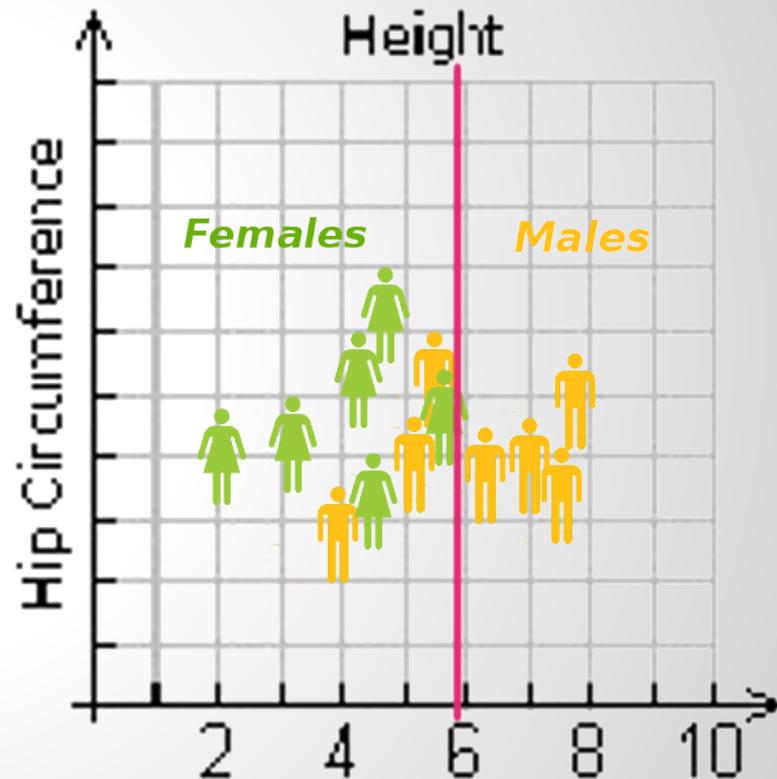
- Classifier which may be only slightly correlated with true classification (accuracy > 50%)
- Examples: Naïve Bayes, logistic regression, decision stumps



# Decision Stumps

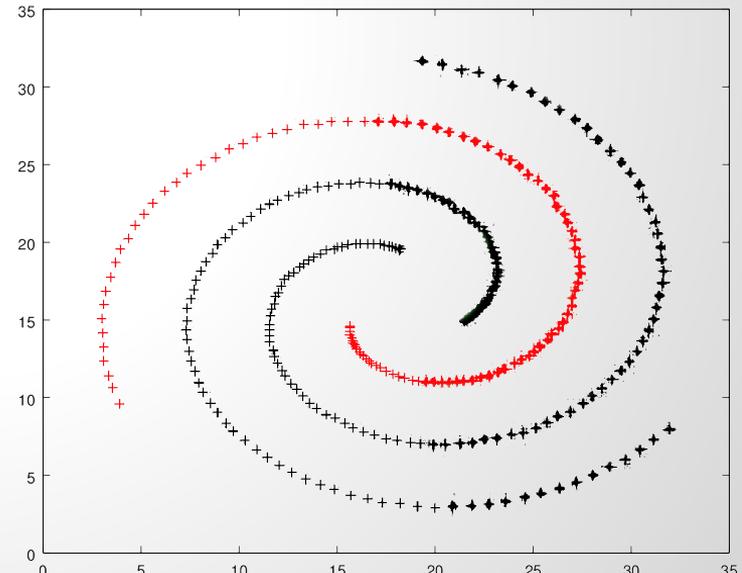
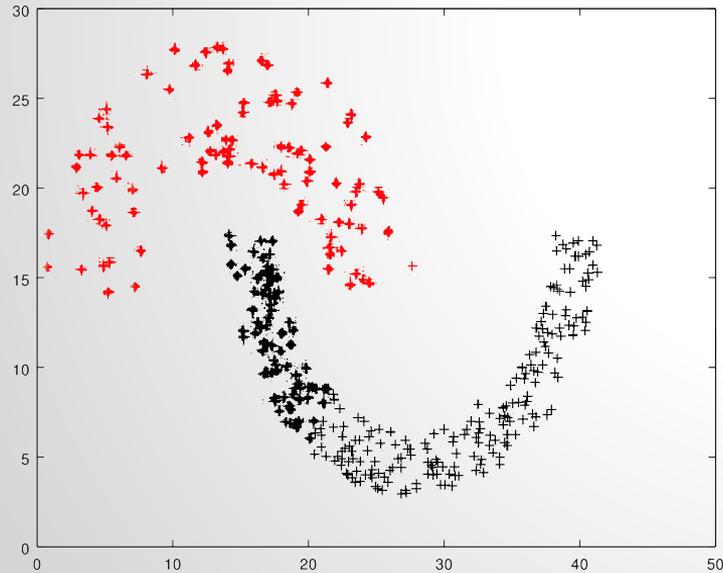


- Single Level Decision Tree
- Focus on a single feature dimension
- Create a decision boundary along that dimension

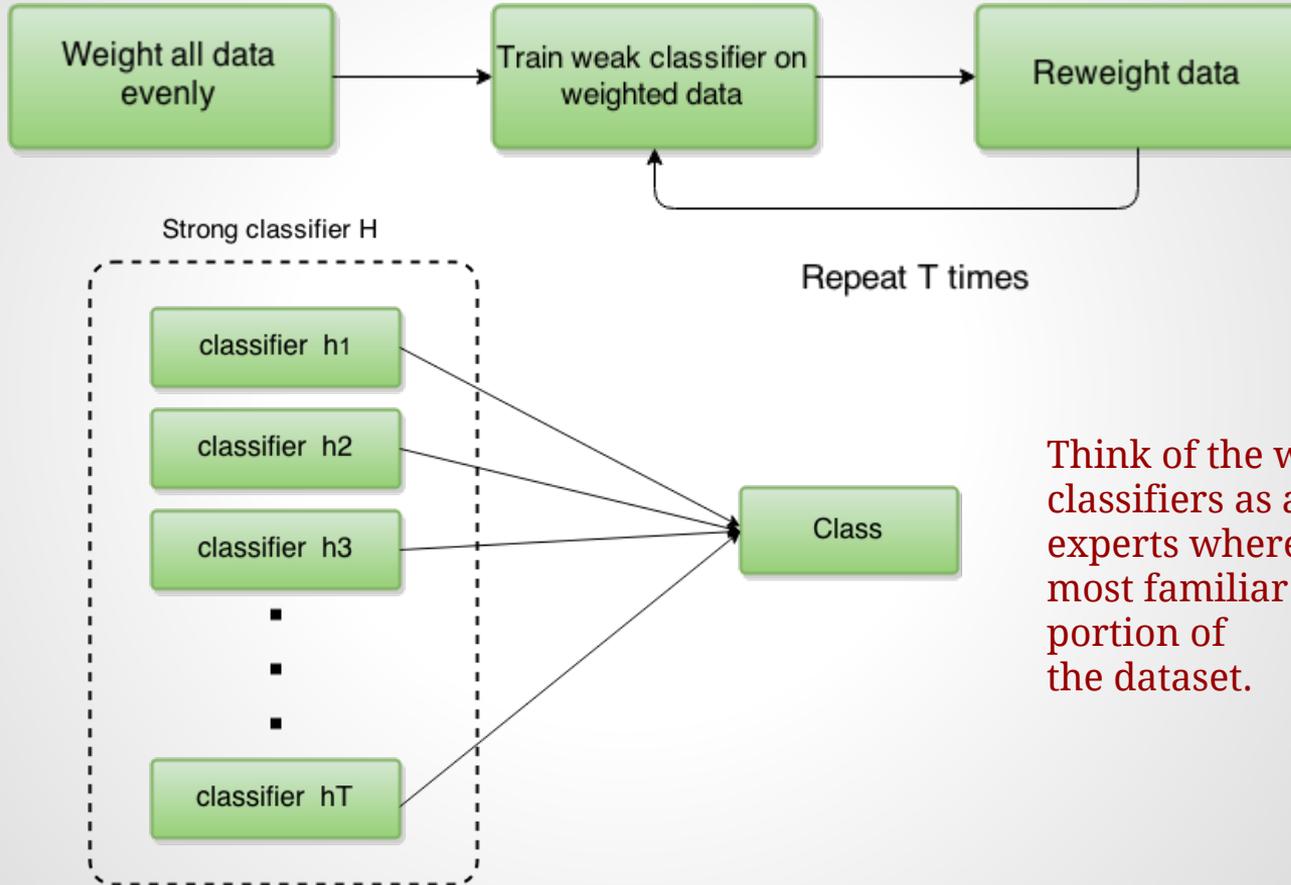


# Advantages of Boosting:

- Easy and fast to train weak classifiers
- Simple models don't usually overfit
- Weak classifiers can not solve hard problems



# Boosting: The Basic Idea



Think of the weak classifiers as a crowd of experts where each is most familiar with some portion of the dataset.

# AdaBoost: Boosting for Binary Classification

Suppose dataset:  $(x_1, y_1), \dots, (x_N, y_N)$

where  $x_i \in \mathbb{R}^n, y_i \in Y = \{-1, 1\}$

Let  $D_t(i) =$  weight of point  $x_i$

**Goal:** Build classifier  $H(x) = \text{sign}(\alpha_1 h_1(x) + \dots + \alpha_T h_T(x))$

where  $h_1(x), \dots, h_T(x)$  are binary classifiers,  
built on distributions  $D_1, \dots, D_T$  respectively.

**Issue:** How to find the best  $\alpha$ 's and  $D$ 's.

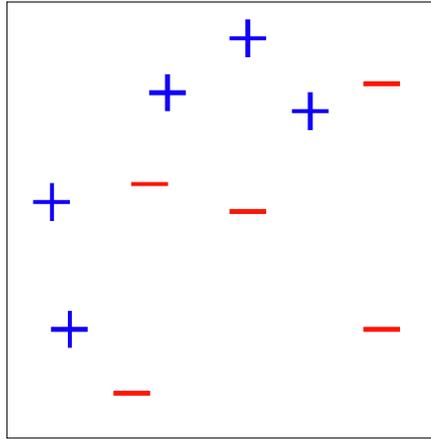
**Answer:** Iteratively minimize exponential loss:

If  $F(x) = \alpha_1 h_1(x) + \dots + \alpha_T h_T(x)$ , then

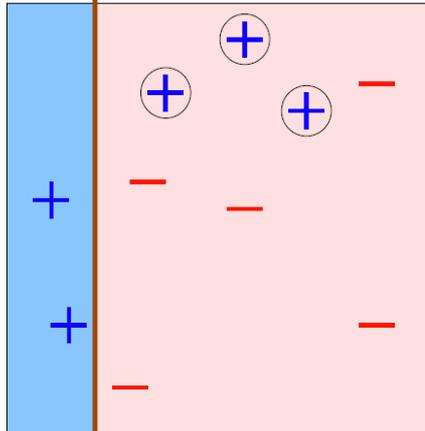
$$L = \frac{1}{N} \sum_{i=1}^T \exp(-y_i F(x_i))$$

# AdaBoost with Decision Stumps as Weak Classifiers

(Shapire, Freund. 1999)



$D_1$



$h_1(x)$

## Round One:

Build  $h_1$  on distribution  $D_1$

Then calculate:

$$\epsilon_1 = Pr_{i \sim D_1}(h_1(x) \neq y_i).$$

(sum of misclassified point weights)

Next calculate  $\alpha_1$ .

Then calculate  $D_2$ .

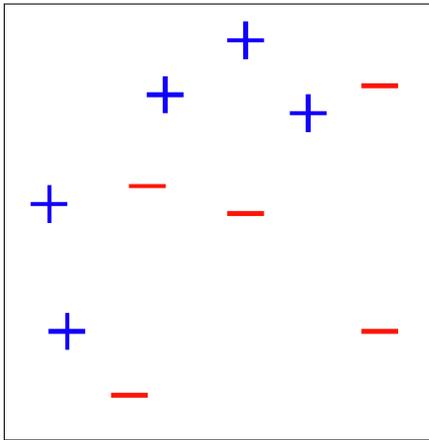
$t = 1, \dots, T$

Train weak classifier  
 $h_t : \mathbb{R}^n \rightarrow R$   
on distrubution  $D_t$

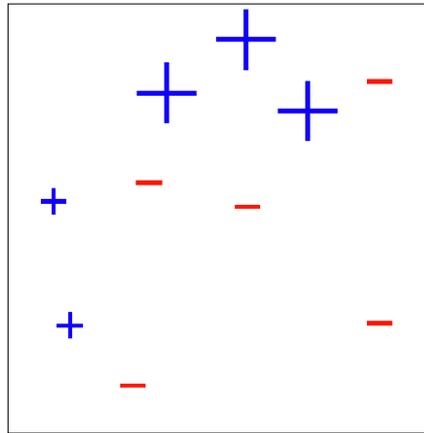
Pick  $\alpha_t$   
(weight for  $h_t$ )  
$$\alpha_t := \frac{1}{2} \ln\left(\frac{1 - \epsilon_t}{\epsilon_t}\right)$$

Set  $D_{t+1}(i) :=$   
$$\frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

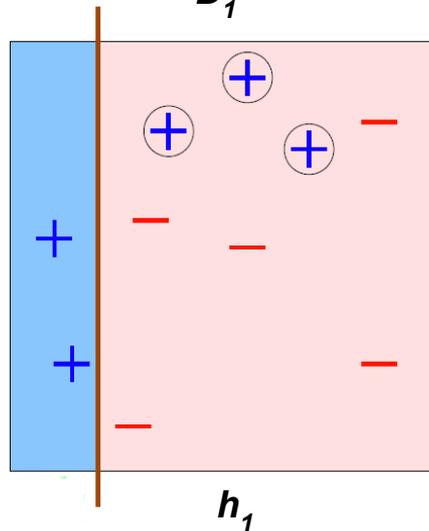
$$H(x) := \text{sign}\left(\sum_{t=1}^T \alpha_t h_t(x)\right)$$



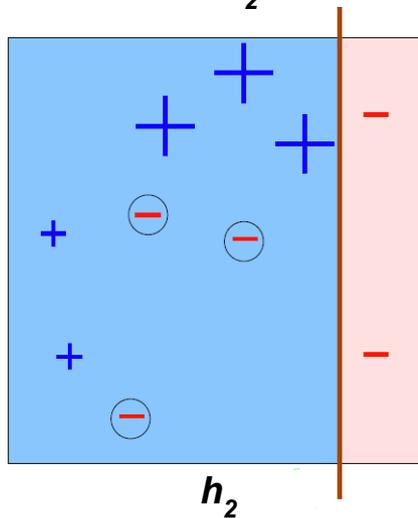
$D_1$



$D_2$



$h_1$



$h_2$

## Round One:

Build  $h_1$  on distribution  $D_1$

$$\varepsilon_1 = 3/10$$

$$\alpha_1 = 0.42$$

$D_2(i) = 0.166$  for  $x_i$  that were misclassified

$D_2(i) = 0.072$  for  $x_i$  that were correctly classified

## Round Two:

Build  $h_2$  on distribution  $D_2$

$$\varepsilon_2 = 0.216$$

$$\alpha_2 = 0.65$$

$$\varepsilon_2 = 0.216, \quad \alpha_2 = 0.65$$

For each  $X_i$  where:

$h_1$  was wrong,  $h_2$  was right:

$$D_3(i) = 0.11, \quad D_2(i) = 0.166$$

$h_1$  was right,  $h_2$  was wrong:

$$D_3(i) = 0.175, \quad D_2(i) = 0.072$$

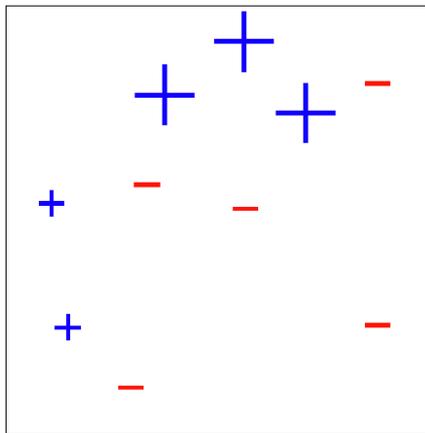
$h_1$  was right,  $h_2$  was right:

$$D_3(i) = 0.047 \quad D_2(i) = 0.072$$

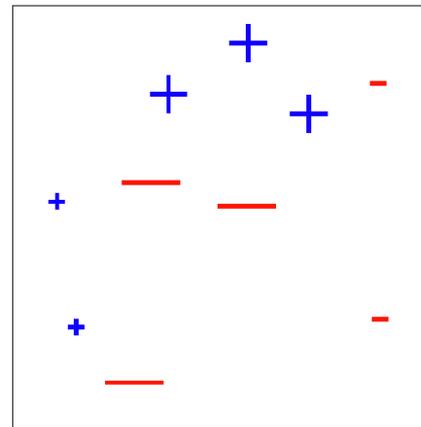
**Round Three:**

Train  $h_3$  on  $D_3$

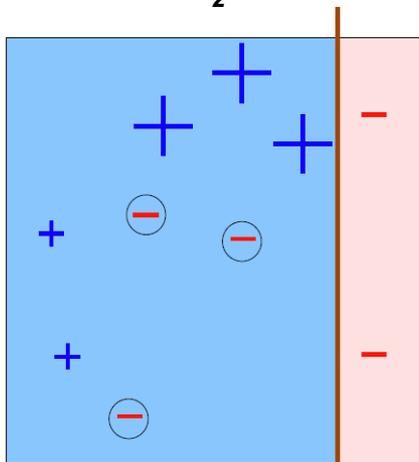
$$\varepsilon_3 = 0.144, \quad \alpha_3 = 0.91$$



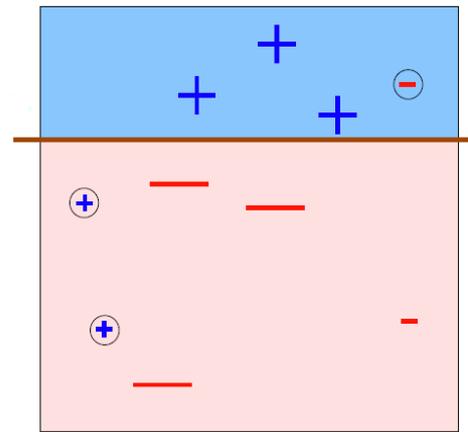
$D_2$



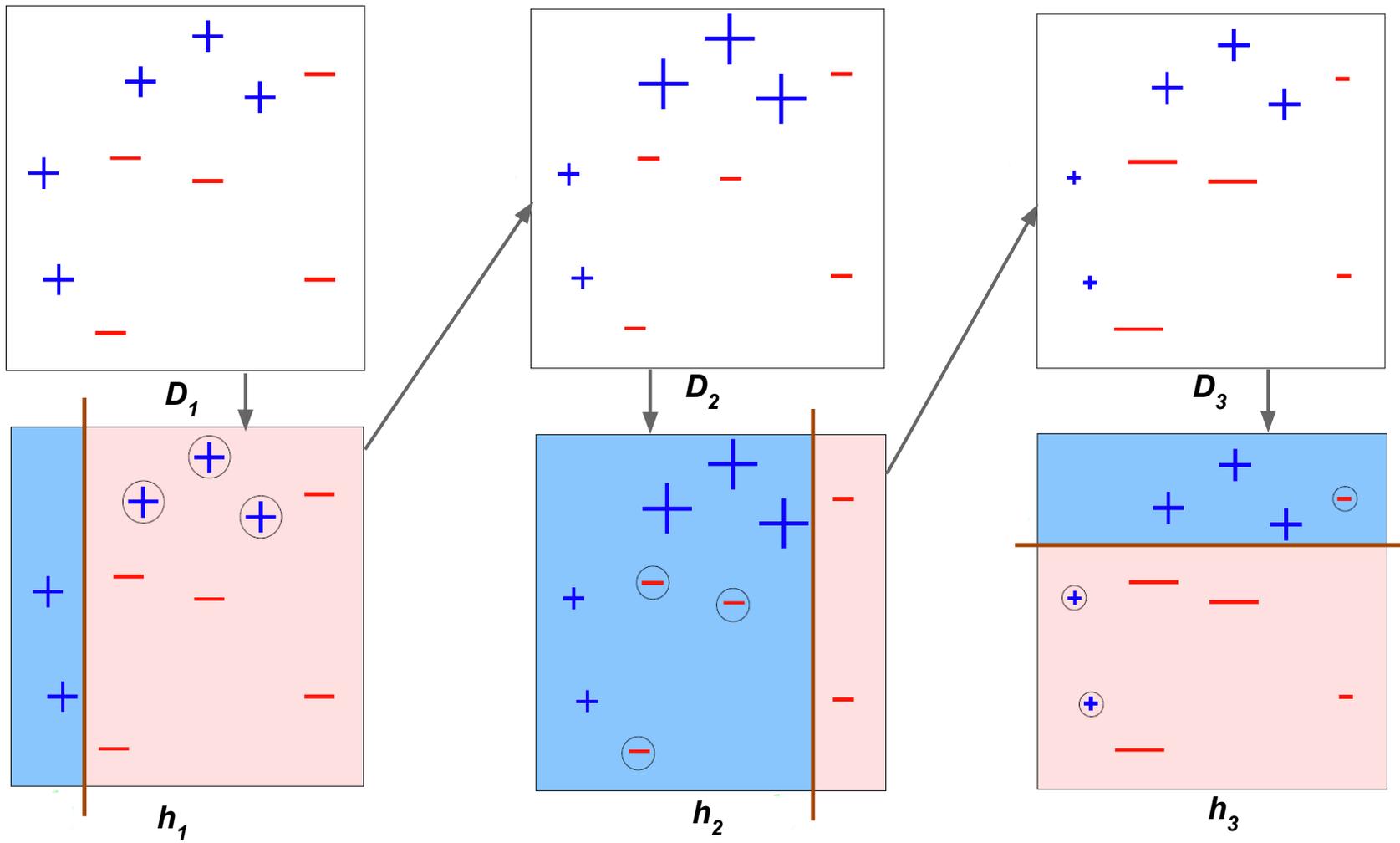
$D_3$



$h_2$



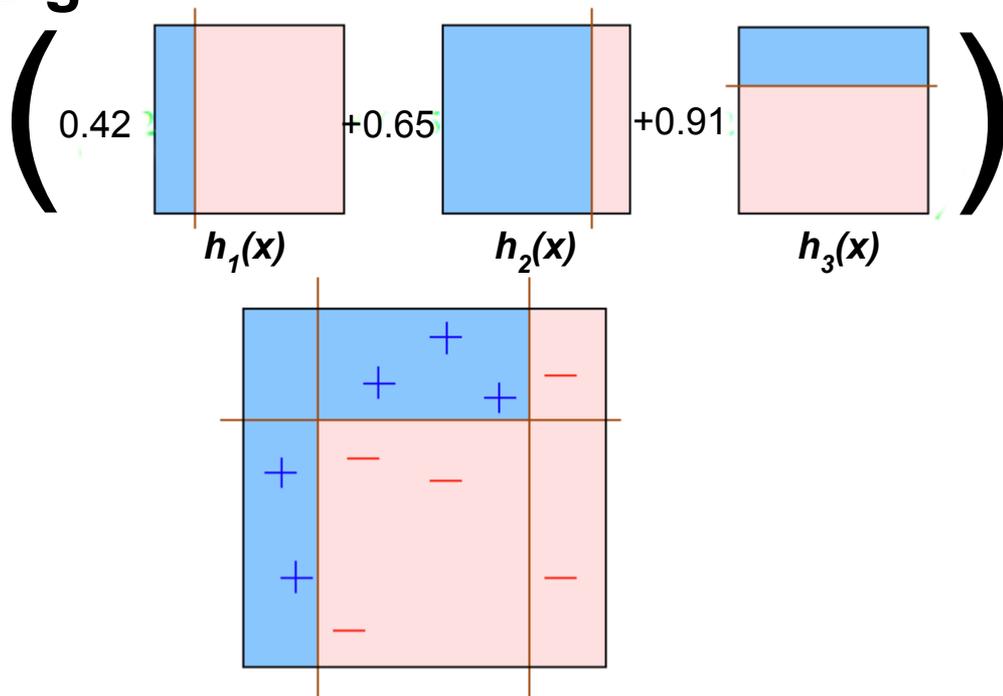
$h_3$



# Strong Classifier

$H(x)$

sign



$t = 1, \dots, T$

Train weak classifier  
 $h_t : \mathbb{R}^n \rightarrow R$   
on distribution  $D_t$

Pick  $\alpha_t$   
(weight for  $h_t$ )

$$\alpha_t := \frac{1}{2} \ln\left(\frac{1 - \epsilon_t}{\epsilon_t}\right)$$

Set  $D_{t+1} :=$

$$\frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$$H(x) := \text{sign}\left(\sum_{t=1}^T \alpha_t h_t(x)\right)$$

# Boosting Demos

Swirly boosting demo

More Swirly boosting demo

AdaBoost in action

# References:

Schapire, R. E. (2003). The boosting approach to machine learning: An overview. In *Nonlinear estimation and classification* (pp. 149-171). Springer New York.

Schapire, R. E. (1990). The strength of weak learnability. *Machine learning*, 5(2), 197-227.

Kearns, M. (1988). Thoughts on hypothesis boosting. *Unpublished manuscript*, 45, 105.

Long, P. M., & Servedio, R. A. (2010). Random classification noise defeats all convex potential boosters. *Machine Learning*, 78(3), 287-304.

Freund, Y., & Schapire, R. E. (1995, January). A decision-theoretic generalization of on-line learning and an application to boosting. In *Computational learning theory* (pp. 23-37). Springer Berlin Heidelberg.

[MIT Boosting Lecture](#)

# Software:

[Wikipedia list from AdaBoost page](#)

[Boosting Song](#)